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A STUDY OF FATIGUE LIFE UNDER RANDOM LOADING

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SUMMARY

Simplified load schedules, based on the statistics obtained from several different methods of analyzing a random-load time history having the properties of band-limited white noise, are used to conduct programed axial-load fatigue tests on 2024-T3 aluminum-alloy edge-notched sheet specimens. Additional tests are conducted using load schedules which vary randomly. A comparison of the test results is made to determine whether or not the fatigue-inducing characteristics of the given random time history are retained when simplified load schedules are used.

An analytical method is developed to generate, digitally, random time histories having arbitrarily shaped power spectra. A programed axial-load fatigue machine has also been developed to conduct tests based on the statistics obtained from the generated time histories. The machine utilizes an IBM punchcard system to apply up to 55 discrete load levels in any given sequence.

The results of the tests indicate that a considerable variation in life is obtained in tests scheduled according to statistics of a given random time history analyzed according to several different counting methods.

INTRODUCTION

The aircraft designer is acutely aware of the structural fatigue problems which exist in modern-day aircraft. In order to design a structure having a reasonable fatigue life he must be able to estimate its life under service loading. Since there are no known theories which can be used to predict accurately the life under service loading, the designer must rely on the fatigue test as a means of obtaining a reliable estimate of life. If the random-load time histories encountered in service could be duplicated in a laboratory test, the estimate obtained from such a test would undoubtedly be considered reliable. However, existing fatigue testing equipment is usually limited to applying simple cyclic loads. Thus, the designer must estimate service life from simple laboratory tests. Several counting methods have been devised for reducing the service loading history to numerical form. These reductions result in several different sets of load statistics that can be used to program fatigue tests which simulate, with varying degrees of complexity, the service loading history. The purpose of the present investigation is to determine whether or not the programed fatigue test, based on the load statistics obtained from the various counting methods, adequately retain the significant fatigue-inducing characteristics of the service loading history.

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In order to determine the significant fatigue-inducing parameters of any random-load time history (not necessarily the service loading history), an analytical method was developed to generate, with the aid of a digital computer, random time histories having arbitrarily shaped power spectra. A digital rather than an analog type of representation was chosen for the following two reasons: (1) the digital time history is in a form directly amenable to reduction by various counting methods and (2) the statistics obtained by the various counting methods can be punched on IBM cards and used as the input to a programed axial-load fatigue testing machine.

This testing machine was developed in order to conduct tests using the load statistics obtained from any digitally generated time history. The machine is capable of applying loads in either a random or programed sequence.

To limit the scope of this paper, only one time history having the properties of band-limited white noise was generated and analyzed according to several different counting methods. The load statistics obtained from these counting methods were used to conduct programed axial-load fatigue tests on 2024-T3 aluminum-alloy edge-notched sheet specimens having a theoretical elastic stress-concentration factor of four. Fatigue life estimates obtained from these tests were compared with estimates obtained from tests conducted using a load schedule which varied randomly. In this manner, it was possible to determine which programed tests preserved the significant fatigue-inducing characteristics of the generated random time history.

GENERATION OF A RANDOM TIME HISTORY

Random numbers were generated, normally distributed, and numerically filtered to obtain a random time history having the properties of band-limited white noise. Of all generators tested, the random number generator selected had the least amount of correlation between numbers. Each random number was considered to be a sample of a continuous random time history taken at discrete uniform intervals of time. For convenience, the time history was normalized to have a unit variance. The following is a description of the procedure used to digitally simulate a random time history having the properties of band-limited white noise. The procedure to be described can be used equally well for digitally simulating other random time histories having arbitrarily shaped power spectra (ref. 1).

Random Number Generator

Random numbers were obtained, using standard digital computer techniques, with a fixed-point pseudo random number generator developed at the National Bureau of Standards (ref. 2). Each generated random number R_N was obtained from the previous random number R_{N-1} by taking the last 11 digits of the product $R_0 R_{N-1}$ where $R_0 = 5^{15}$ and $N = 1, 2, 3, \ldots$. Numbers were then selected at random from the generated R_N. Only the first six digits p of the randomly selected 11-digit number were used in this investigation. Approximately 320,000 random numbers were obtained in this manner, each having an equally likely chance of occurring (i.e., uniform probability distribution) where $0 \leqq p \leqq 999999$.

Transformation to Normal Distribution

The random numbers were transformed into a normal distribution with mean equal to zero and unit variance by an approximate formula developed by Tukey (ref. 3). This was done in order to simulate a stationary, Gaussian random process. The equation required that the random numbers p be between zero and one. Therefore, all numbers, p, were divided by 10^6 and designated q. The expression is

$$X = 4.91\left[q^{0.14} - (1 - q)^{0.14}\right]$$

where q is the normalized random number and X is the normal deviate. In the present investigation, it was found that when X became greater than 2.4 there were significant departures from the normal distribution. Hence, it was necessary to make an additional correction to the equation as follows:

$$X \approx X' = 4.91\left[q^{0.14} - (1 - q)^{0.14}\right]$$

$$X = X' \quad \text{when} \quad |X'| \leqq 2.4$$

$$X = |X'| + 0.13\left(|X'| - 2.4\right)^2 \quad \text{when} \quad |X'| > 2.4$$

where the sign of X takes the sign of X'. It should be noted that all normal deviates fall in the range $-4.91 \leqq X \leqq 4.91$.

It is assumed that the normally distributed random numbers represent a sampling of a continuous time history $y(t)$ at discrete uniform intervals of time $\Delta t = 1/2$ second. Consequently, the random numbers can be thought of as discrete set of values y_i equal to $y(t)$ when $t = i\Delta t$ and undefined in between. There is no loss of information when making the above assumption if $y(t)$ contains no frequencies greater than the Nyquist or folding frequency, f_F (ref. 4) where

$$f_F = \frac{1}{2\Delta t}$$

If frequencies higher than f_F are present in the data they will contribute energy or power to the lower frequencies which will result in errors in the power spectrum. In order to avoid this situation, the higher frequencies should be removed by appropriate filtering.

It is also assumed that the time history $y(t)$ is stationary (i.e., statistical properties are invariant with time) and Gaussian in nature.

Numerical Filtering of Random Numbers

If the random process $y(t)$ is passed through a linear system having an amplitude or frequency response, $F\left(\frac{f}{f_F}\right)$, a new random process $Y(t)$ will be obtained. Thus

$$Y(t) = F\left(\frac{f}{f_F}\right)y(t) \tag{1}$$

The frequency response $F\left(\frac{f}{f_F}\right)$ can be defined such that the new random process $Y(t)$ will have a specified power spectrum, provided that $y(t)$ is stationary and Gaussian and the system is linear. This can be done by utilizing the input-output relation of power spectral analysis which states that the product of the input power spectrum $\varphi_{in}(f)$ and the square of the absolute value of the frequency response $\left|F\left(\frac{f}{f_F}\right)\right|^2$ (also called a transfer function) is equal to the output power spectrum $\varphi_{out}(f)$. Thus

$$\varphi_{out}(f) = \left|F\left(\frac{f}{f_F}\right)\right|^2 \varphi_{in}(f) \tag{2}$$

The frequency response, $F\left(\frac{f}{f_F}\right)$, can be determined from this equation since $\varphi_{in}(f)$ can be calculated (using the relationship between the autocorrelation function and power spectral density, ref. 4) and $\varphi_{out}(f)$ is the specified power spectrum, in this case band-limited white noise. Knowing $F\left(\frac{f}{f_F}\right)$, the new time history $Y(t)$ can now be calculated using equation (1).

In order to perform this operation numerically, the original time history $y(t)$ is replaced by the discrete set of values $y_{(i+K)}$ and the frequency-response function $F\left(\frac{f}{f_F}\right)$ by a set of filter factors a_K. The new time history $Y(t)$ having the desired shaped power spectrum is represented by a discrete set of values Y_i which are determined as follows:

$$Y_i = \sum_{K=-M}^{M} a_K y_{(i+K)}$$

Details for computing the filter factors a_K, which were obtained by representing the frequency-response function with a Fourier cosine series, are given in appendix A. A sample of the filtered time history obtained in this manner is shown in figure 1. For clarity, the values Y_i have not been plotted but rather the faired curve through these values. The input-output relationship used to obtain this time history is also shown in figure 1.

Analysis of Random Time History

The digitally generated random time history described in the previous paragraphs has been analyzed according to several different counting methods. The results obtained are several sets of statistics which describe, with varying degrees of complexity, the fluctuations encountered in the generated time history. The statistics are obtained by counting the number of times a certain event occurs throughout the random time history. An event may be considered to be a peak value (positive or negative), a distance between peak values (a range), or the crossing of a particular level by the time history in either the positive or negative direction. Events are counted in each of 50 equal intervals over the range -5.0 to +5.0 which encompasses the maximum and minimum values of the random numbers generated. The time scale becomes totally irrelevant since the rate at which the time history fluctuates does not enter into the counting. In addition, the results of the various counting methods do not normally preserve the sequence of occurrence of events. However, special procedures were employed to retain the sequence for three conditions in this investigation.

The counting procedures to be used in this investigation have been previously discussed by Schijve (ref. 5). A brief description of each counting method used plus some of the counting results on a time history having the properties of band-limited white noise and consisting of 320,000 random numbers are presented in the following paragraphs.

Peak count method.- This method provides a count of all the positive peak values and all the negative peak values occurring in each of the 50 intervals of $Y(t)$. The peak values counted are indicated in the upper portion of figure 2. Also presented in this figure are the frequency distributions of the positive and negative peaks. Peaks counted in a given interval of $Y(t)$ as indicated in the figure are plotted at the center of the interval. It should be noted that the distributions of positive peaks and negative peaks are symmetrical about zero which is a good indication that the generated time history is truly random. Another method of counting the peaks is to count only the positive peaks above zero and negative peaks below zero. The results for this latter method of counting can be obtained from figure 2 by eliminating the positive peaks occurring below zero and the negative peaks occurring above zero. A modification of the latter method is frequently used for counting maneuver loads encountered in a service load time history of a fighter-type aircraft.

Maximum peaks between zero crossings.- As indicated in figure 3, only the highest peaks between zero crossings are counted. This counting procedure eliminates fluctuations of considerable size as can be seen by comparing the frequency distribution of the maximum peaks between zero crossings (fig. 3) with the distribution of all the peaks (fig. 2). This method is similar to the method generally used for counting gust loads encountered in a service load time history of a transport-type aircraft.

Amplitude regardless of mean.- An amplitude is defined as half the range between successive peaks. It is considered to be positive when the slope of the curve between successive peaks is positive and negative when the slope is negative. This counting method gives no information about the peak values. The values counted are indicated in figure 4 along with the results of the counting.

Level crossings.- This counting method and the following method afford other means of obtaining peak distributions which are most frequently used to program fatigue tests. This method records the number of times the time history crosses a given level with positive slope. It is logical to expect that the reference or zero axis of the time history will be crossed most frequently with progressively fewer crossings occurring at the higher or lower levels. The number of peaks occurring in the interval between two adjacent levels (not necessarily the true number of peaks in the interval) was obtained by subtracting the respective number of crossings at each of these levels. Figure 5 shows the distribution of peaks obtained in this manner. It should be noted that there are relatively few peaks in the intervals close to zero. This can be explained by the fact that positive and negative peaks in the same interval cancel each other, leaving only positive or negative peaks in the interval. (See appendix B.) The same result can be obtained by subtracting, point by point, the two curves in figure 2. The curve for negative peaks reduces the curve for positive peaks significantly in the intervals near zero and vice versa.

Level crossings eliminating small fluctuations.- This counting procedure is similar to the level crossing count method but must satisfy a secondary condition. Level crossings (positive slope) are counted only if the trace of the time history crosses some lower preset value a distance Δ below the level to be counted. In counting level crossings below the reference or zero axis (negative slope) the procedure is reversed, that is, level crossings are counted only if the trace crosses some preset value a distance Δ above the level to be counted. Level crossing of the zero axis can be counted using either a positive or negative slope. For this particular investigation, a value of $\Delta = 0.4$ was selected. This increment corresponds to a stress interval in the testing program which is below the fatigue limit and thus insignificant with respect to fatigue. In essence, the preassigned value of Δ eliminates from the counting results all fluctuations in the time history having ranges (peak-to-peak) less than the value Δ. The results of the counting are presented in figure 6. As for level crossings, the peaks (again not necessarily the true number of peaks) were obtained by subtracting the number of crossings at adjacent levels. A comparison of the distribution of peaks obtained in this manner with that of the distribution of peaks obtained by level crossings will show the number of fluctuations with ranges less than 0.4 that have been eliminated. It should be remembered that occurrences in an interval are plotted at the midpoint of the interval.

Means and amplitudes.- This method of counting provides data that are generally regarded as most significant in fatigue. A mean is defined as one-half the algebraic sum of two adjacent peak values and an amplitude as one-half the algebraic difference between the same two adjacent peak values. This method provides a count of all the amplitudes about a corresponding mean value over the entire range of mean values. The counting method is illustrated in figure 7 and three typical distributions of amplitudes and their corresponding means are presented. It should be noted that the distributions retain essentially the same shape regardless of the mean value selected. It should also be pointed out that this counting method can be used to obtain the results of some of the other counting methods, that is, peaks and amplitudes regardless of mean.

Means and amplitudes eliminating small fluctuations.- This counting method is similar to the mean-amplitude count method in that it counts the mean and amplitude between adjacent peak values. However, it was decided to eliminate all fluctuations having ranges less than 0.4 for the same reasons as discussed previously. At places in the time history where these small fluctuations occur (range 0.4) a mean and amplitude between the peaks immediately before and immediately after these small fluctuations are obtained. Means and amplitudes for fluctuations with ranges greater than 0.4 are counted in the usual manner. The count is made as shown in figure 8 and three typical distributions of amplitudes and their corresponding mean values are presented. A comparison with figure 7 will show the number of fluctuations less than 0.4 that have been eliminated.

In the following paragraphs the statistics obtained by the above counting methods will be utilized to develop programed load fatigue tests which simulate, with various degrees of complexity, the completely random-load fatigue test. It should be noted that an attempt has been made to determine some of the above statistics from the power spectrum of a random time history (ref. 1).

TEST PROCEDURES

Specimens

Sheet specimens of 2024-T3 aluminum alloy were used for this investigation. The specimens contained edge notches which produced a theoretical elastic stress concentration $K_T = 4$. Figure 9 shows the specimen details. The material for these specimens is part of a stock of nominal 0.090-inch-thick 2024-T3 aluminum-alloy sheet retained in the Langley Research Center for fatigue test purposes.

Testing Machine

A servo-hydraulic machine was developed for use in this investigation. A "block" diagram of the machine is shown in figure 10. The loading frame has a nominal capacity of ±20,000 pounds in axial load. Cycling rates up to 7 cps are obtainable, depending on the load range. The important features of this programed load fatigue machine are: (1) any type of load history defined by discrete load levels can be programed in any arbitrary sequence utilizing punched IBM cards; (2) 55 individually adjustable input channels, each being identified by its own code, can be preset to any value between zero and full scale; and (3) a high degree of load accuracy is maintained throughout the test.

In operation, the card reader transmits coded load information to the switching logic. Upon receipt of this information, the switching logic performs selected functional checks, then switches the desired input channel (preset 10-turn potentiometer) into the servo-loop summing point. At the summing point, the voltage from the input channel is combined with the output from a load-sensing strain-gage bridge mounted on the weigh-bar which is in series with the specimen. The combined voltage from the summing point is fed into the carrier amplifier where the incoming voltage is compared with a reference voltage to determine the magnitude and polarity of the signal sent to the d-c amplifier. In the following discussion the signal sent to the d-c amplifier shall be called error which has both magnitude and polarity. The d-c amplifier uses the error to cause the servo valve to direct oil to one side of the ram thus loading the specimen and thereby changing the output from the strain-gage bridge. The hydraulic oil flow is proportioned by the magnitude of the error signal, thus as the load applied to the specimen approaches the desired load (error approaches zero) the flow is slowed so that the load approaches the desired load asymptotically. The minimum acceptable error is controlled by the settings of the load comparator. When the load error has been reduced to a value equal to or less than the comparator settings, the load comparator generates a signal which commands the reader to transmit the next piece of load information to the switching logic.

A circuit is provided which checks continuously for malfunction in any portion of the loading system. Specimen failure (complete separation of the specimen), the detection of an error in the switching logic functional checks, or loss of command signal to the servo-valve cause an immediate stop in oil flow by deenergizing the solenoid-controlled flow valve.

Loads are monitored by either a strip chart recorder or a null indicating a-c bridge. The strip chart is used to scan for extraneous loads, whereas the a-c bridge is used for static load measurements and to check system damping. The whole system is calibrated periodically and static indication is repeatable to 0.1 percent of full scale. True load accuracy is estimated to be ±2 percent of full scale.

Three of these programed load machines were used in this investigation.

Load Schedules

Load schedules were developed utilizing the statistical results of each of the previously described counting methods. In developing these load schedules, the following assumptions were made: (1) mean stress = 17.4 ksi (this corresponds to zero on the arbitrary scale used to "count" the time history) and (2) maximum stress = 43.5 ksi (this corresponds to +5.0 on the arbitrary scale). A description of the load schedules developed for each of three general types of tests follows:

Random tests.- For load programs using a random distribution of peaks, the loads were programed using 50 levels. These levels correspond to the 50 levels in the arbitrary scale for counting the time history. Three load programs were developed with a random distribution of peaks; these were (1) original generated time history, (2) maximum peaks between zero crossings, and (3) means and amplitudes eliminating small fluctuations ($\Delta = 0.4$). For these three load programs the sequence of load levels was preserved from the original generated time history. The fatigue life obtained in tests using the original generated time history will be used as the basis for determining whether or not a given simplification to the original time history retained the essential fatigue-inducing characteristics of the generated time history.

Eight-step constant mean block tests.- Five load programs were developed with eight load steps and a constant mean. Because the distributions of negative and positive peaks were found to be approximately equal, only the positive distributions were used to develop load schedules. In all of the tests with eight steps and a constant mean each positive load was followed by an equal negative load. For these load programs the range of peaks between zero and maximum was divided into eight equal bands. Each band was then represented by a discrete load level at the middle of the band. The number of times each discrete load level was applied is defined by the intercepts of the band limits. One time through the distribution is defined as a "block" with the "block" being approximately 4,000 load cycles for each counting method. Within a block, all of the cycles of load at each load level were applied consecutively with the order of load levels being varied randomly according to a schedule taken from a table of random numbers. The sequence of load levels was different for each block until the twentieth block; thereafter, the schedule for the first 20 blocks was repeated. The sequence of load levels was the same for all tests. Eight-step constant mean block tests were conducted utilizing the results of the following counting methods: (1) maximum peaks between zero crossings, (2) amplitudes regardless of mean, (3) level crossings, (4) level crossings eliminating small fluctuations, and (5) peak count (positive peaks >0).

In (4) above, a value of $\Delta = 0.4$ was selected because this would eliminate stress cycles which were below the fatigue limit of the specimen. This is a reasonable approach because no damage is expected from stress cycles which have a maximum value less than the fatigue limit. (See ref. 6.)

Eight-step variable mean block tests.- The results of the amplitudes and means and amplitudes and means eliminating small fluctuations counting methods are obtained as distributions of amplitudes about various means. In order to develop load programs based on these statistics, the distributions of amplitudes for several means were grouped together into a combined distribution of amplitudes, about a fixed mean. For example, the distribution of amplitudes with means of -0.3 -0.1, +0.1, and +0.3 were combined into a distribution of amplitudes with a 0 mean. Similarly, distributions were obtained for three positive means and three negative means. The distribution for 0 mean was divided into eight equal amplitude bands in both the positive and negative direction. The representative amplitude for each band was selected in the middle of the band.

This then defines the values of 16 load setting potentiometers. To facilitate testing procedures, the remaining combinations of means and amplitudes were selected so that they could be applied by the proper sequencing of these 16 settings. Figure 11 shows schematically how this was accomplished.

As in the case of the constant mean block tests each positive amplitude was followed by an equal negative amplitude. Also, the total number of cycles was approximately 4,000 per block and the sequence of load combinations within a block was random.

Tests were conducted in this manner using the statistics obtained from the following counting methods: (1) means and amplitudes, and (2) means and amplitudes eliminating small fluctuation.

DISCUSSIONS OF RESULTS

General

The results of 60 variable-amplitude axial-load fatigue tests conducted on sheet specimens of 2024-T3 aluminum alloy are presented in figure 12. In figure 12, the symbols represent the geometric mean of six tests conducted using the same load program. The scatter in a given set of tests never exceeded $1\frac{1}{2}$ to 1 and was generally less than $1\frac{1}{3}$ to 1. This trend is in agreement with other variable-amplitude tests conducted at the NASA Langley Research Center (refs. 6, 7, and 8.)

Basis for Comparison

The fatigue life obtained in random tests employing the original generated time history is assumed to be the life which would have been obtained in service. This life is compared with the life obtained by utilizing the results of a simplifying counting procedure. In this way an evaluation can be made of the ability of the counting procedures to preserve the fatigue-inducing characteristics of the generated time history.

The results of all the tests were normalized on the results of tests conducted using the original generated time history (random). This was done by dividing the original time history into equal segments, each segment containing

5,000 random numbers. The average number of peaks per 5,000 numbers, for each of the different counting methods employed, was determined. The equivalent number of segments of the original time history required for failure was determined by dividing the average number of peaks for a given counting method by the total number of peaks required for failure, utilizing the same counting method. The number of equivalent segments obtained in this manner, for each of the counting methods employed, was compared to the number of segments required for failure utilizing the original time history.

The results of the variable-amplitude tests are summarized in the following table:

Test condition	Normalized life	Remarks
Random		
Peak count method	1.0	Generated time history
Maximum peak between zero crossing	1.10	See note 1
Means and amplitudes eliminating small fluctuations	1.01	See note 2
Eight-step, constant mean, block tests		
Peak count method	0.97	See note 3
Maximum peak between zero crossings	1.26	See note 1
Amplitudes regardless of mean	2.28	See note 4
Level crossings	1.07	See note 2
Level crossings eliminating small fluctuations	1.18	See note 2
Eight-step, variable mean, block tests		
Means and amplitudes	0.63	See note 5
Means and amplitudes eliminating small fluctuations	0.63	See note 5

Note 1. The maximum peaks between zero crossings method eliminates approximately 25 percent of the peaks in a given segment of generated time history. The magnitude of these peaks can be reasonably large and their elimination would be expected to increase fatigue life. The difference between results obtained in the random tests and the eight-step constant mean, block tests is probably due to an

order effect in the block tests. Tests based on this counting method appear to lead to longer lives than the random time histories they are to represent.

Note 2. These counting methods, in general, eliminate small fluctuations near the zero mean, which have been shown to have a very small effect on fatigue life. (See ref. 6.)

Note 3. A shorter normalized life was anticipated. In these tests, positive peaks were treated as though each occurred between zero crossings, thus making this type of test more damaging than the generated time history.

Note 4. When the amplitude is retained but the mean discarded, the damaging effect of mean stress variations is reduced and therefore a longer fatigue life results. This method of service load simulation may lead to grossly unconservative answers.

Note 5. This result was very surprising. The reason for this short life is not obvious although the deleterious effect of the mean stress probably is an important factor.

Examination of the results obtained in this investigation leads to the general conclusion that the majority of the counting procedures employed do successfully retain the essential fatigue characteristics of the particular time history investigated. However, it is important to note that the answers obtained in this investigation were for one type of load history and one series of tests for which certain assumptions were made. It is probable that different answers would have been obtained if (1) the generated time history had vastly different statistical properties than the one used, (2) a different range of stress values had been used, or (3) different assumptions had been made for such variables as block size, location of the representative amplitude within the amplitude band, and number of stress levels to represent a given distribution. The effects of some of these variables on fatigue life are discussed in references 6, 7, and 8.

CONCLUDING REMARKS

A method has been presented for evaluating the ability of several counting procedures to retain the essential fatigue-inducing characteristics of a given random time history. A digitally generated random time history having an arbitrarily shaped power spectrum (in this case band-limited white noise), was analyzed using several different counting methods and the results obtained were used to program axial-load fatigue tests on notched-sheet specimens.

The results of the fatigue tests were normalized on the life obtained in tests using the generated time history (random) and ranged from 0.63 for block tests using 38 combinations of means and amplitudes to 2.28 for block tests in which all amplitudes were applied about a common mean. Although correlation was very good for some cases, many additional tests are needed on time histories having different power spectra before general conclusions can be made as to the ability of a given counting method to retain the fatigue-inducing characteristics of a time history.

APPENDIX A

DETERMINATION OF COEFFICIENTS OF A FOURIER COSINE SERIES

REPRESENTATION OF FREQUENCY-RESPONSE FUNCTION

Consider the continuous periodic function of time, of amplitude A and frequency f, such that

$$y(t) = A \cos 2\pi ft$$

If this is sampled at discrete time intervals Δt, the continuous time history is replaced by a discrete set of values y_i equal to $y(t)$ when $t = i\Delta t$ and undefined in between. Thus

$$y(t) = y_i = A \cos 2\pi fi\Delta t \quad \text{for} \quad t = i\Delta t$$

$$y_i = A \cos 2\pi fi\Delta t \frac{f_F}{f_F} \quad \text{where} \quad f_F = \frac{1}{2\Delta t}$$

$$y_i = A \cos 2\pi \frac{f}{f_F} \frac{i\Delta t}{2\Delta t}$$

$$y_i = A \cos i\pi \frac{f}{f_F}$$

The above time history can be modified to change its frequency characteristics (i.e., numerically filtered) as follows:

$$Y_i = \sum_{K=-M}^{M} a_K y_{(i+K)} \tag{A1}$$

where

Y_i filtered time history

$y_{(i+K)}$ original time history

a_K filter factors or coefficients

Equation (A1) represents, in numerical form, the passage of an input signal $y(t)$, through some linear system $F\left(\frac{f}{f_F}\right)$, which results in an output signal $Y(t)$ (ref. 4). Thus

$$Y(t) = y(t)F\left(\frac{f}{f_F}\right) \tag{A2}$$

Upon substituting for $y_{(i+K)}$

$$Y_i = A\left[a_{-M} \cos \pi \frac{f}{f_F}(i - M) \ . \ . \ . + a_{-K} \cos \pi \frac{f}{f_F}(i - K) \ . \ . \ . \right.$$

$$+ a_0 \cos \pi \frac{f}{f_F} i \ . \ . \ . + a_K \cos \pi \frac{f}{f_F}(i + K) \ . \ . \ .$$

$$\left. + a_M \cos \pi \frac{f}{f_F}(i + M)\right]$$

$$Y_i = A\left\{a_0 \cos \pi \frac{f}{f_F} i + \sum_{K=1}^{M}\left[a_{-K} \cos \pi \frac{f}{f_F}(i - K) + a_K \cos \pi \frac{f}{f_F}(i + K)\right]\right\}$$

$$Y_i = A \cos \pi \frac{f}{f_F} i\left(a_0 + 2\sum_{K=1}^{M} a_K \cos \pi \frac{f}{f_F} K\right)$$

$$Y_i = y_i\left(a_0 + 2\sum_{K=1}^{M} a_K \cos \pi \frac{f}{f_F} K\right) \tag{A3}$$

The term in parentheses is in the form of a Fourier cosine series. It is therefore necessary to represent the frequency-response function $F \frac{f}{f_F}$ in the form of a Fourier cosine series in order to filter the generated time history. This was done as follows:

let

$$\frac{f}{f_F} = \frac{h}{H}$$

where $h = 0, 1, 2, \ldots H$, then

$$F\left(\frac{f}{f_F}\right) \rightarrow F\left(\frac{h}{H}\right) = F_h = \frac{A_0}{2} + \sum_{h=1}^{H} A_n \cos\frac{\pi nh}{H}$$

where

$$A_n = \frac{2}{H}\int_0^H F_h \cos\frac{\pi nh}{H}\, dh$$

Using the trapezoidal rule of numerical integration

$$\left\{A_n\right\} = \frac{2}{H}\left[\cos\frac{\pi nh}{H}\right]\left[I_T\right]\left\{F_h\right\}$$

where

$$\left[I_T\right] = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 1 & & & & \\ 0 & & 1 & & & \\ \cdot & & & \cdot & & \\ \cdot & & & & 1 & \\ \cdot & & & & & \frac{1}{2} \end{bmatrix}$$

solving for $\left\{F_h\right\}$

$$\left\{F_h\right\} = \frac{H}{2}\left[I_T\right]^{-1}\left[\cos\frac{\pi nh}{H}\right]^{-1}\left\{A_n\right\}$$

Define

$$\left[\cos\frac{\pi nh}{H}\right]^{-1} = \frac{2}{H}\left[I_T\right]\left[\cos\frac{\pi nh}{H}\right]\left[I_T\right]$$

then

$$\left\{F_h\right\} = \left[\cos\frac{\pi nh}{H}\right]\left[I_T\right]\left\{A_n\right\}$$

therefore

$$F_h = \frac{A_0}{2} + \sum_{h=1}^{H-1} A_n \cos\frac{\pi nh}{H} + \frac{A_H}{2}(-1)^h$$

Finally the coefficients A_n are related to the filter factors a_K as follows:

$$a_0 + 2\sum_{K=1}^{M} a_K \cos \pi \frac{f}{f_F} K = \frac{A_0}{2} + \sum_{h=1}^{H-1} A_n \cos\frac{\pi nh}{H} + \frac{A_H}{2}(-1)^h$$

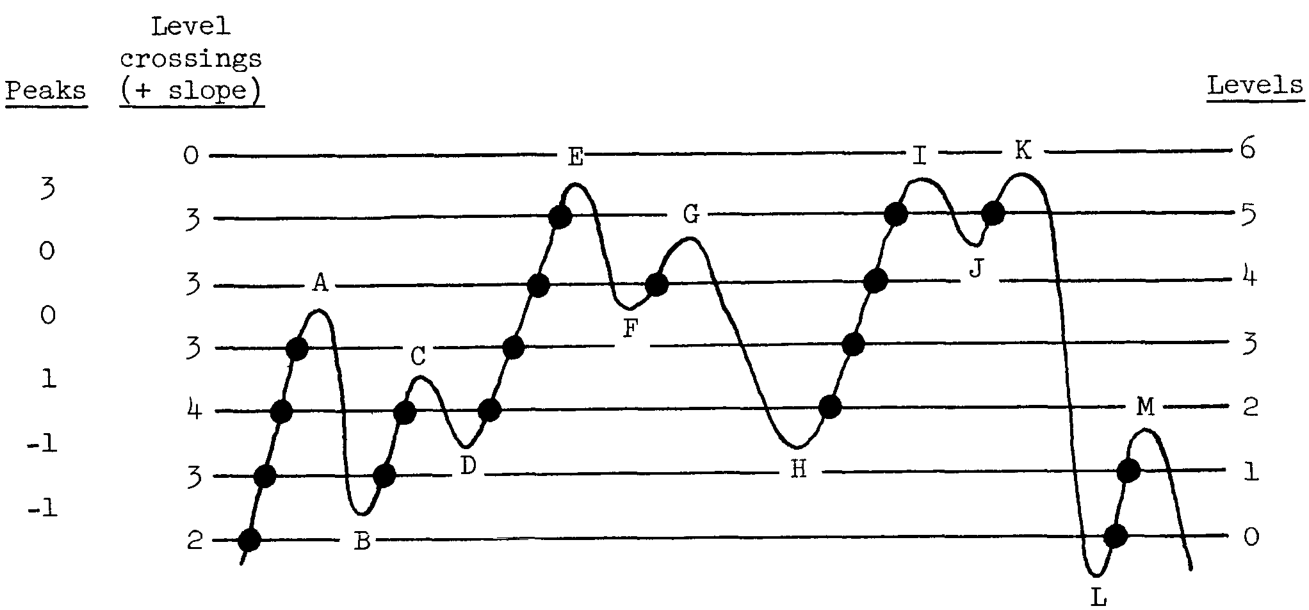
and from equating coefficients

$$a_0 = \frac{A_0}{2}, \quad a_1 = \frac{A_1}{2}, \quad a_2 = \frac{A_2}{2}, \quad \ldots, \quad a_h = \frac{A_h}{2}, \quad a_H = \frac{A_H}{4}$$

APPENDIX B

PEAKS OBTAINED FROM LEVEL CROSSING COUNT METHOD

As mentioned in the text, the level crossing count method is one of several methods used to obtain a peak count. This method does not, however, provide an accurate count of peak values particularly near the mean or reference axis of the time history. The reason cited was that negative and positive peaks in a given interval tend to cancel each other leaving only positive or negative peaks in the interval. The following sketch will illustrate that positive and negative peaks in a given interval do cancel each other resulting in an erroneous peak count.



Points A, C, E, G, I, K, and M are considered to be positive peaks, while B, D, F, H, J, and L are considered to be negative peaks. Counting level crossings with positive slope, the results are as indicated to the left of the time history. Peaks are then obtained by subtracting the number of level crossings at adjacent levels and are indicated at the left of the time history. It should be remembered that, of the two adjacent levels the higher level is subtracted from the lower level. This procedure will determine whether the peaks in the interval are positive or negative. When the peaks are visually counted in the intervals, the following values are obtained and are compared with the peak count from the level crossing method.

Interval	6-5		5-4		4-3		3-2		2-1		1-0	
Peaks	+	-	+	-	+	-	+	-	+	-	+	-
Visual	3	0	1	1	1	1	1	0	1	2	0	1
Level crossings	3	0	0	0	0	0	1	0	0	1	0	1

In intervals 5-4 and 4-3 a positive and a negative peak occur but are not counted by the level crossing method. In the interval 2-1 two negative peaks and one positive peak occur but the level crossing method only counts one negative peak for that interval. Thus, it appears that positive and negative peaks in a given interval cancel each other leaving only positive or negative peaks in the interval. Therefore, the level crossing method does not give a true indication of the peaks.

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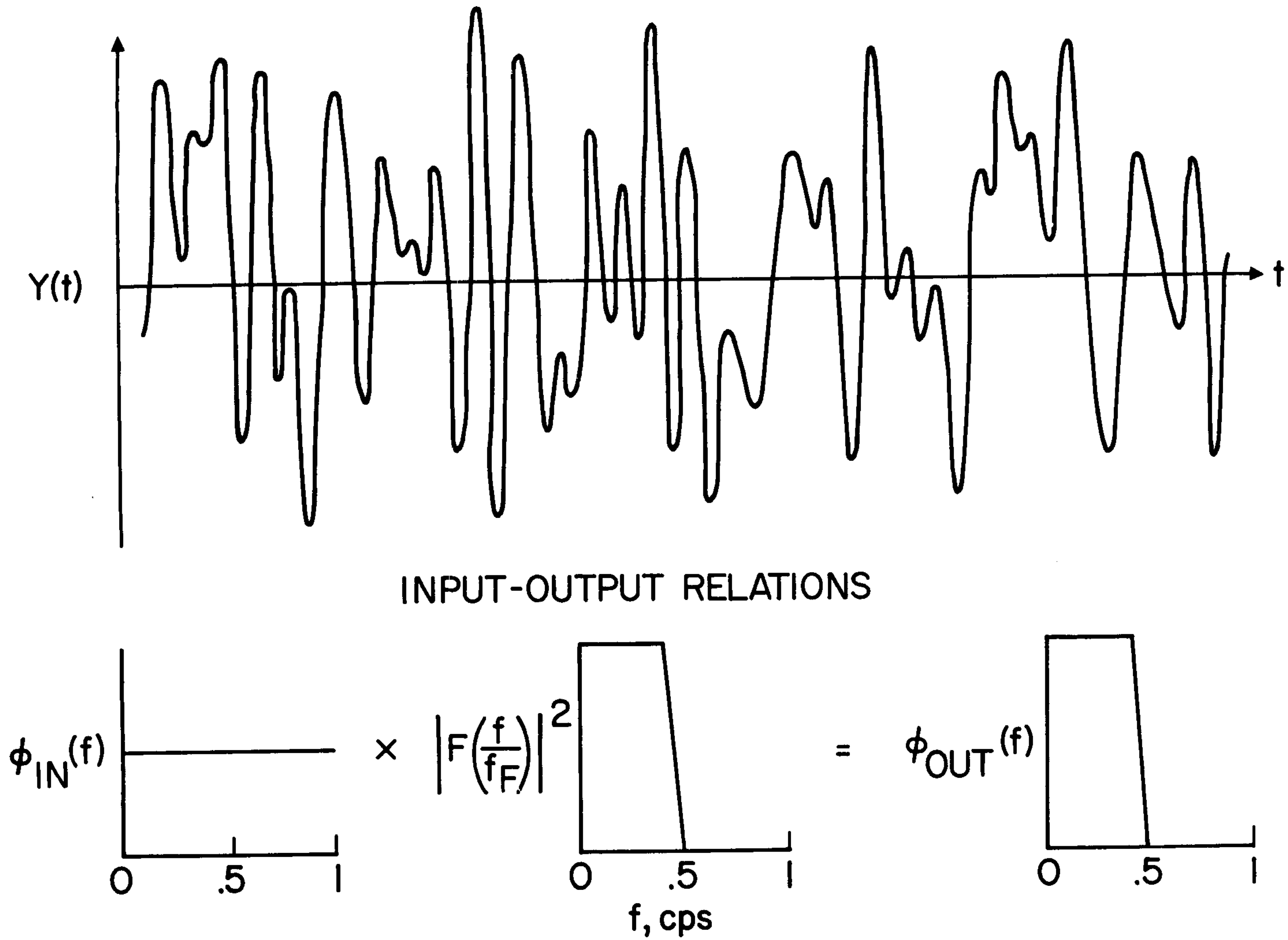

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Figure 1.- Sample of filtered time history and input-output relations of power spectral analysis.

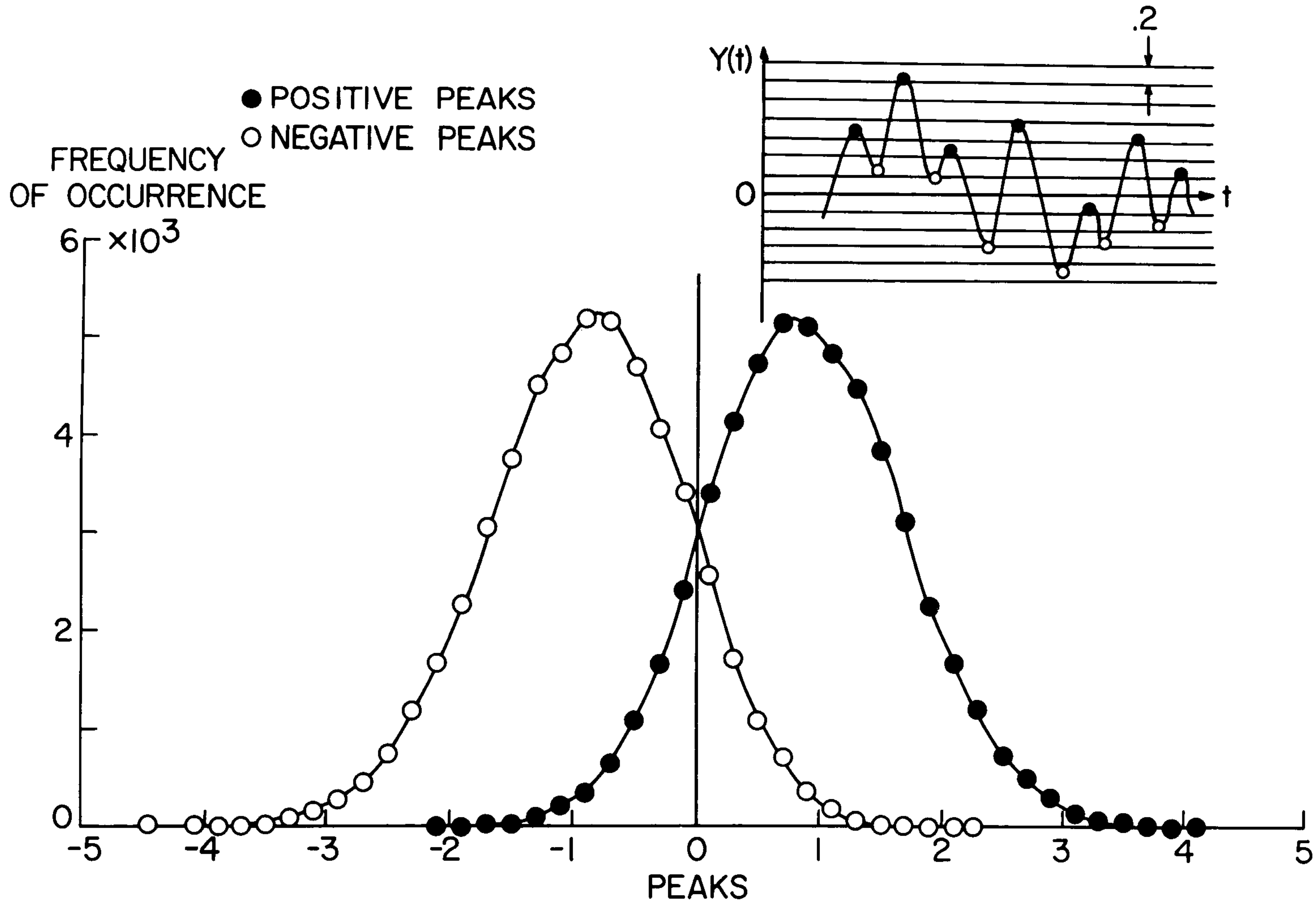

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Figure 2.- Distributions of positive and negative peaks, band-limited white noise consisting of 320,000 random numbers.

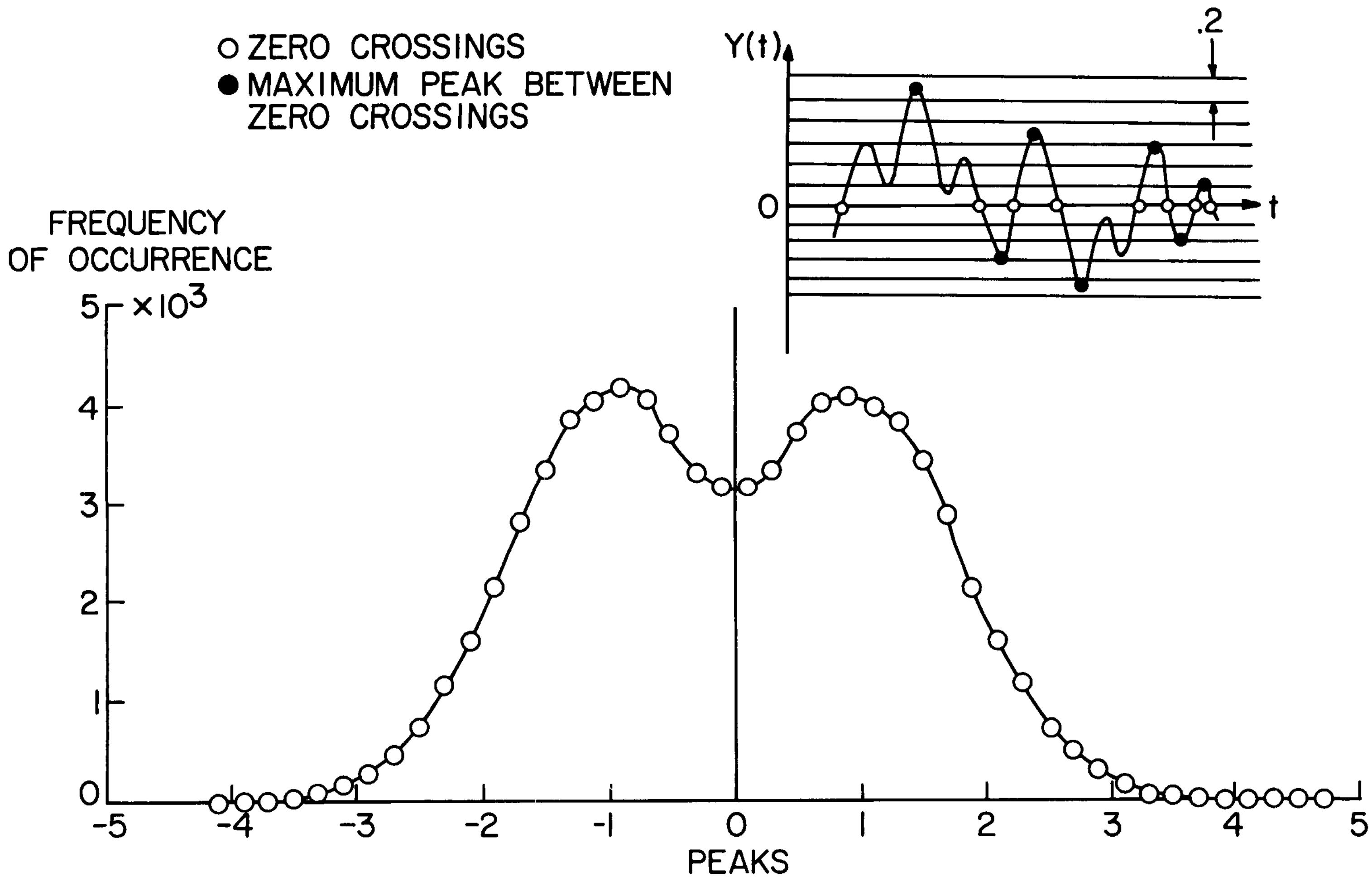

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Figure 3.- Distribution of maximum positive and negative peaks between zero crossings, band-limited white noise consisting of 320,000 random numbers.

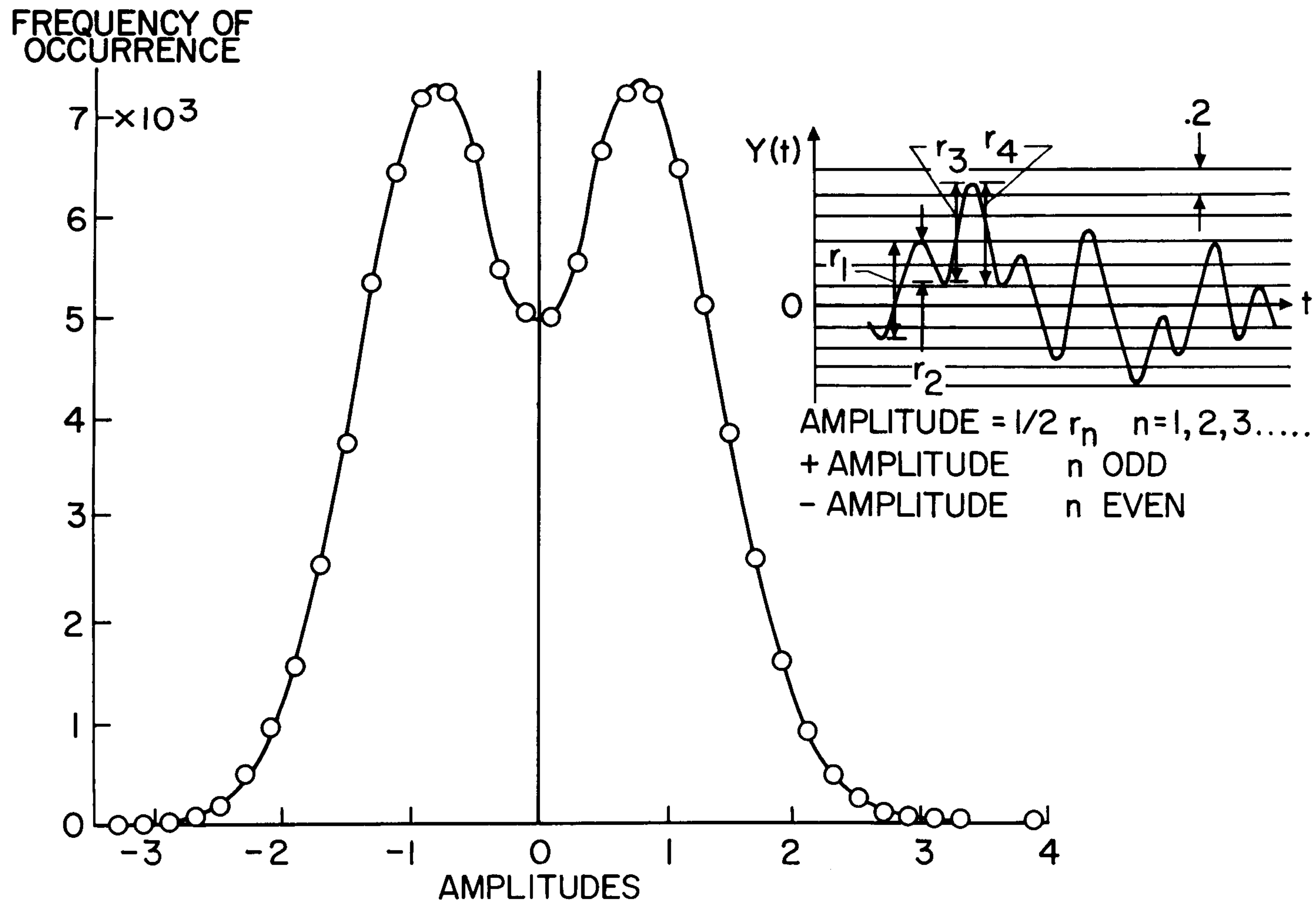

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Figure 4.- Distribution of positive and negative amplitudes regardless of mean, band-limited white noise consisting of 320,000 random numbers.

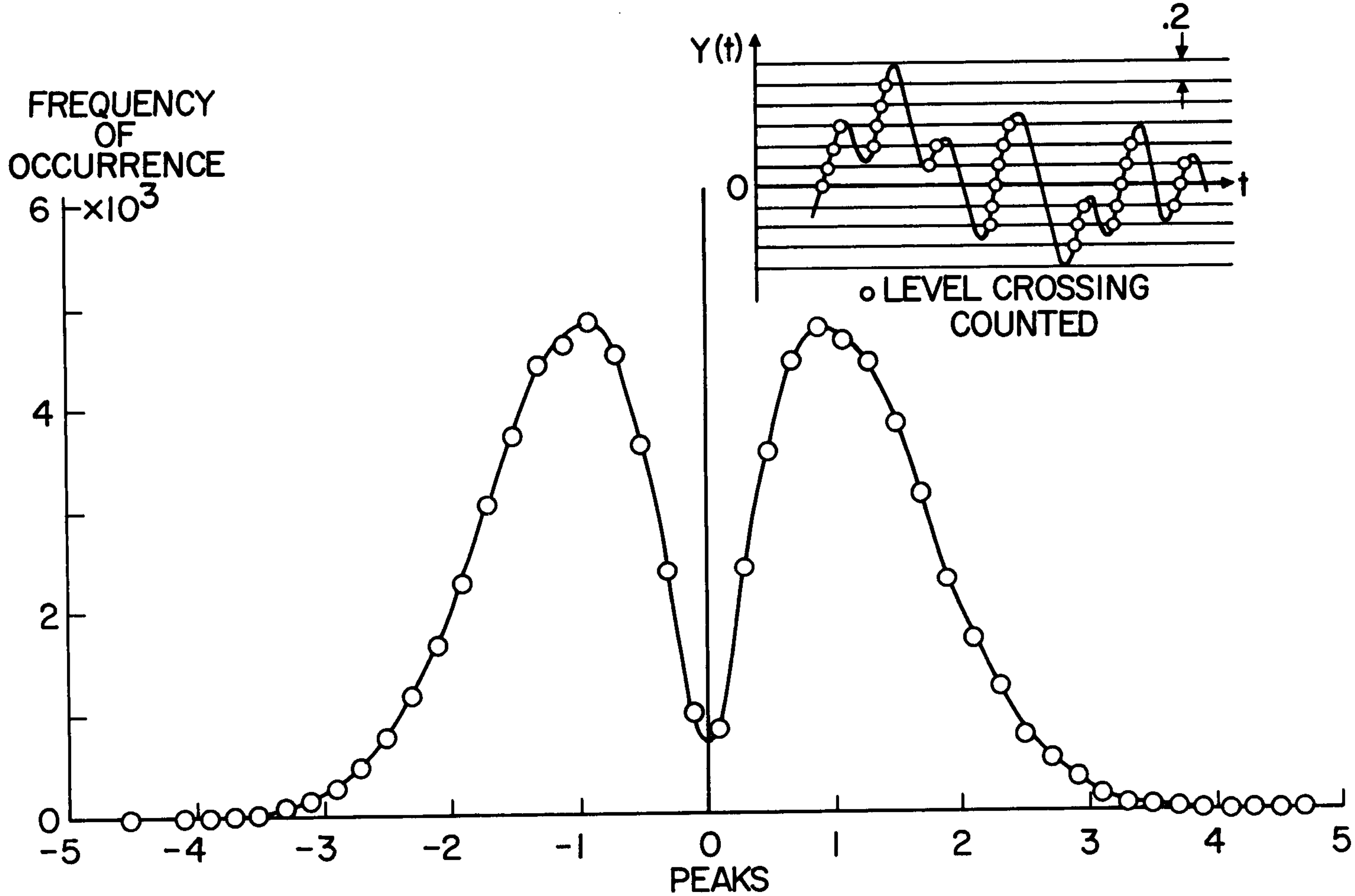

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Figure 5.- Distribution of positive and negative peaks as determined by the level crossing method, band-limited white noise consisting of 320,000 random numbers.

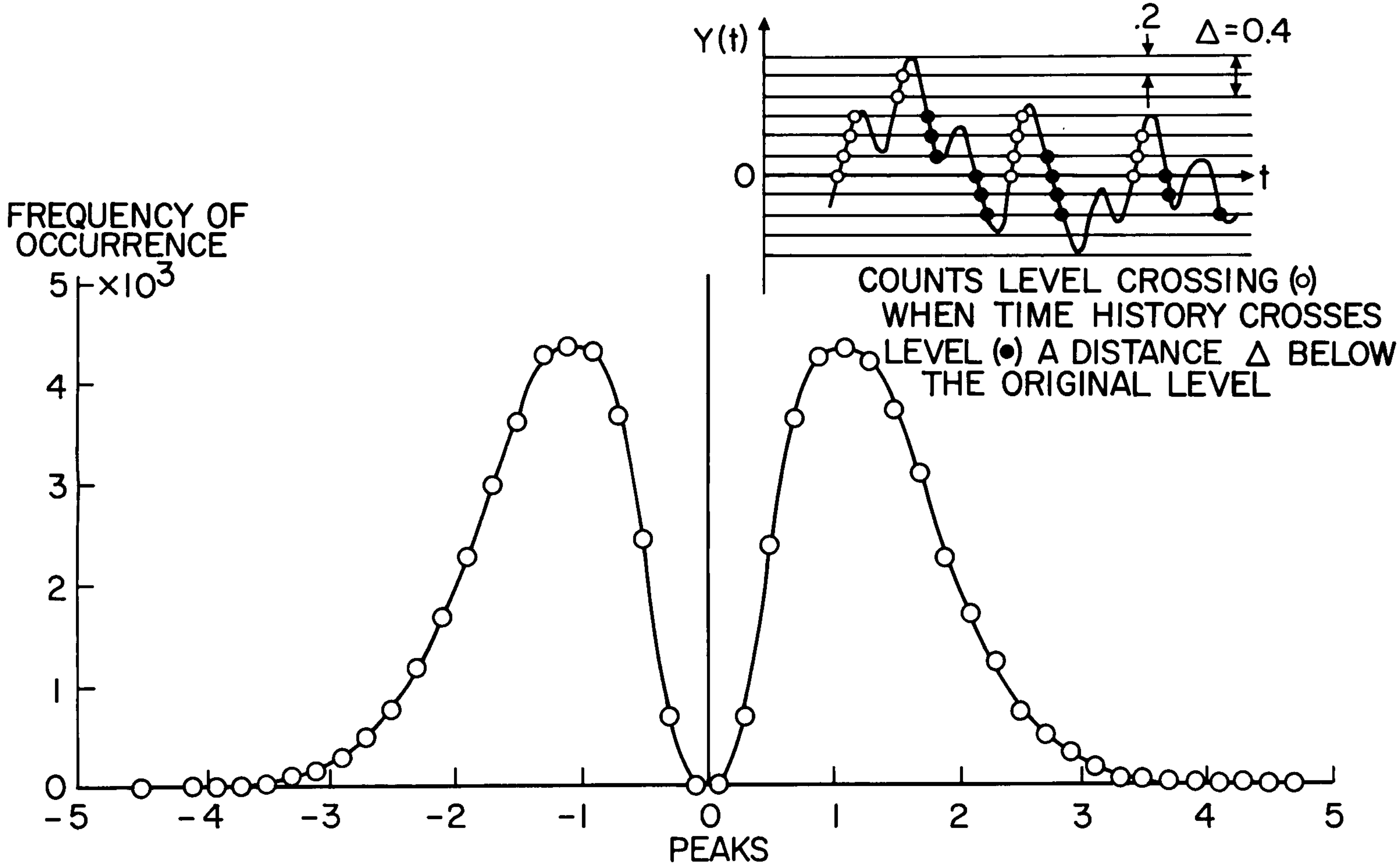

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Figure 6.- Distributions of positive and negative peaks as determined by the level crossings eliminating small fluctuations method, band-limited white noise consisting of 320,000 random numbers.

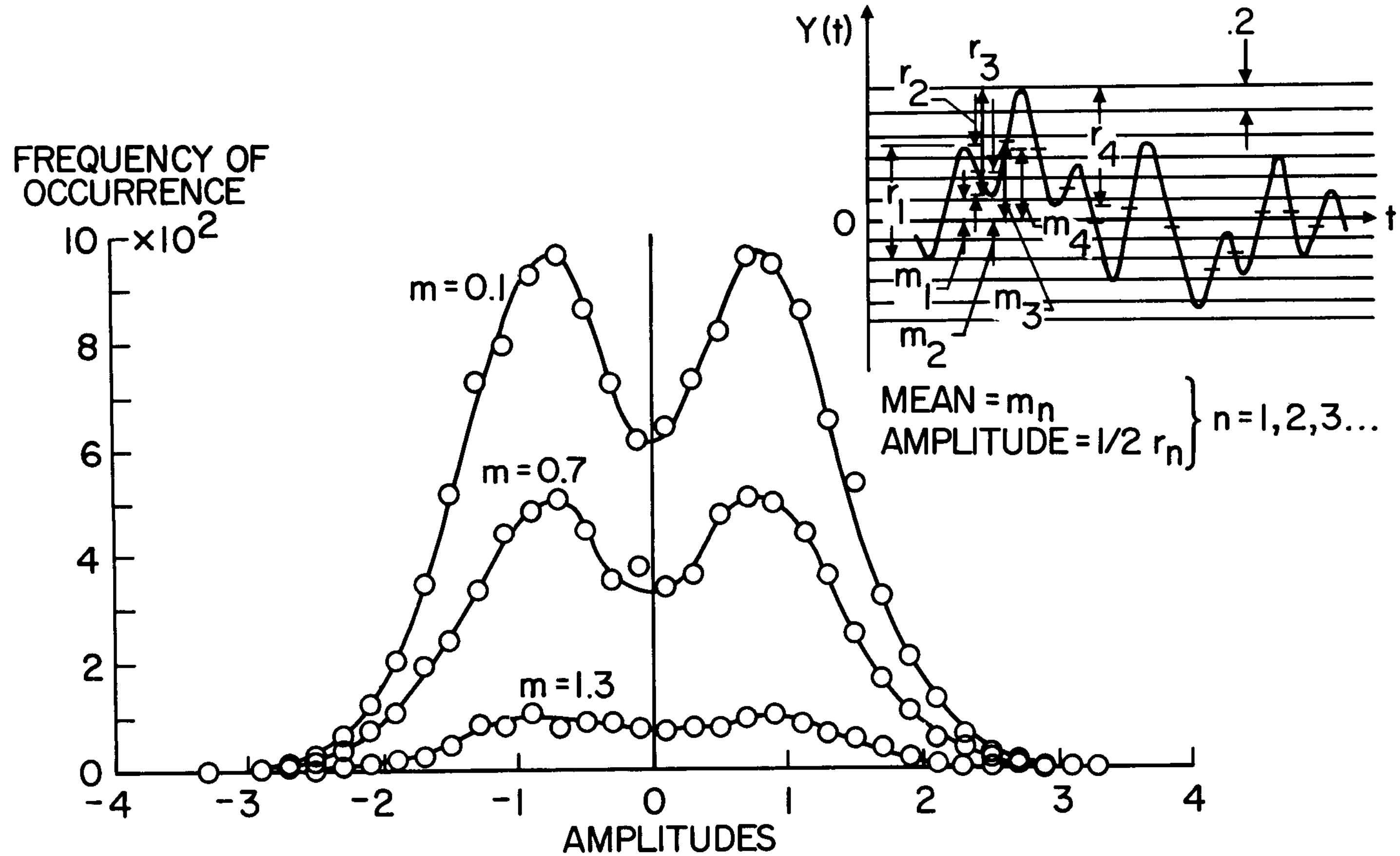

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Figure 7.- Distributions of positive and negative amplitudes about a specified mean, band-limited white noise consisting of 320,000 random numbers.

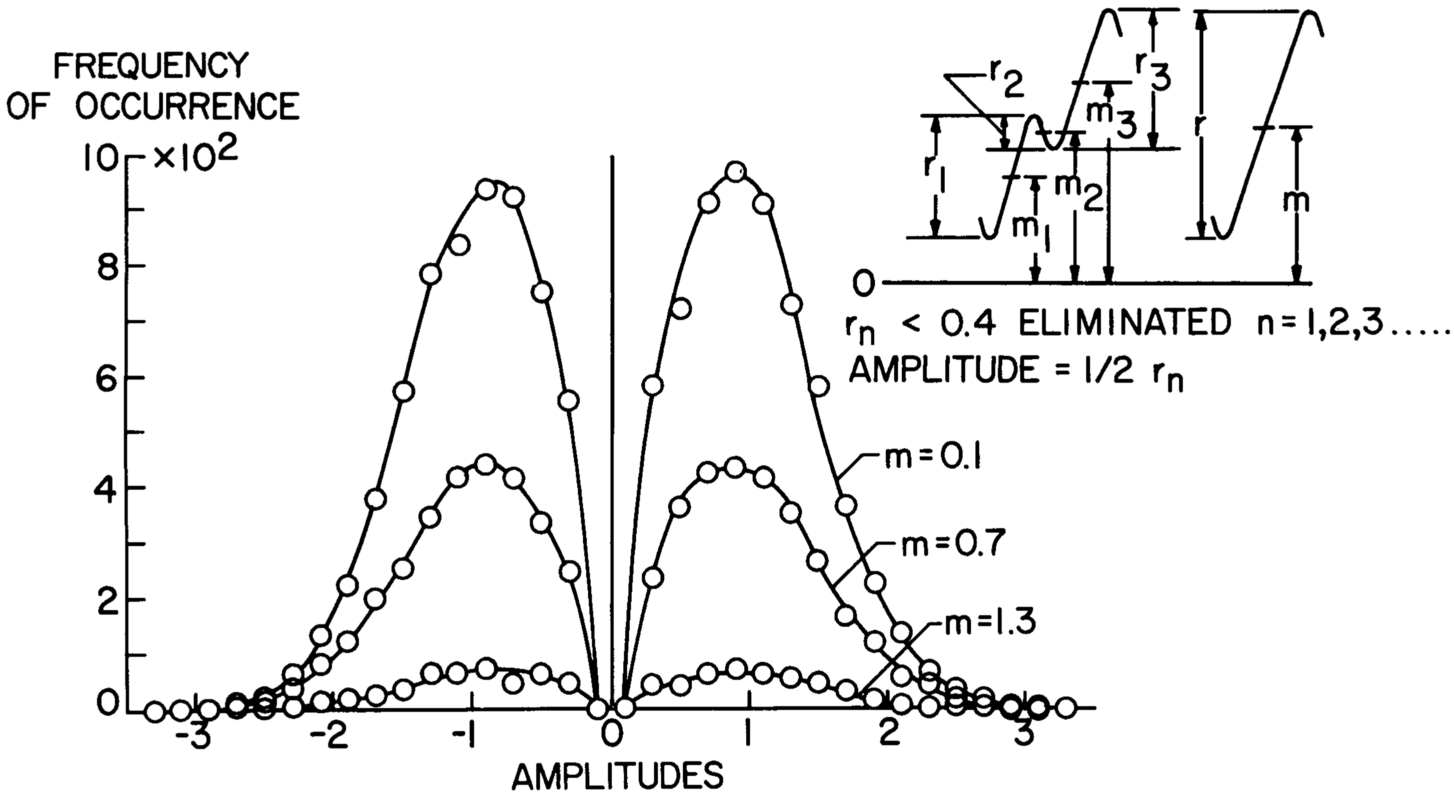

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Figure 8.- Distributions of positive and negative amplitudes about a specified mean (ranges < 0.4 eliminated), band-limited white noise consisting of 320,000 random numbers.

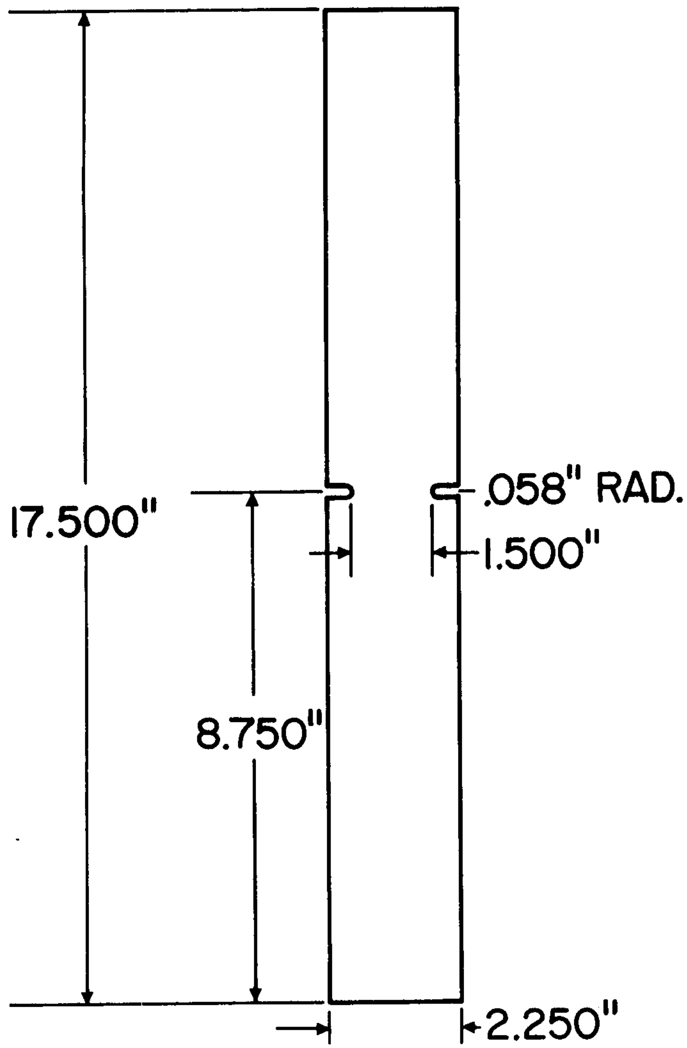

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Figure 9.- Sheet-specimen details.

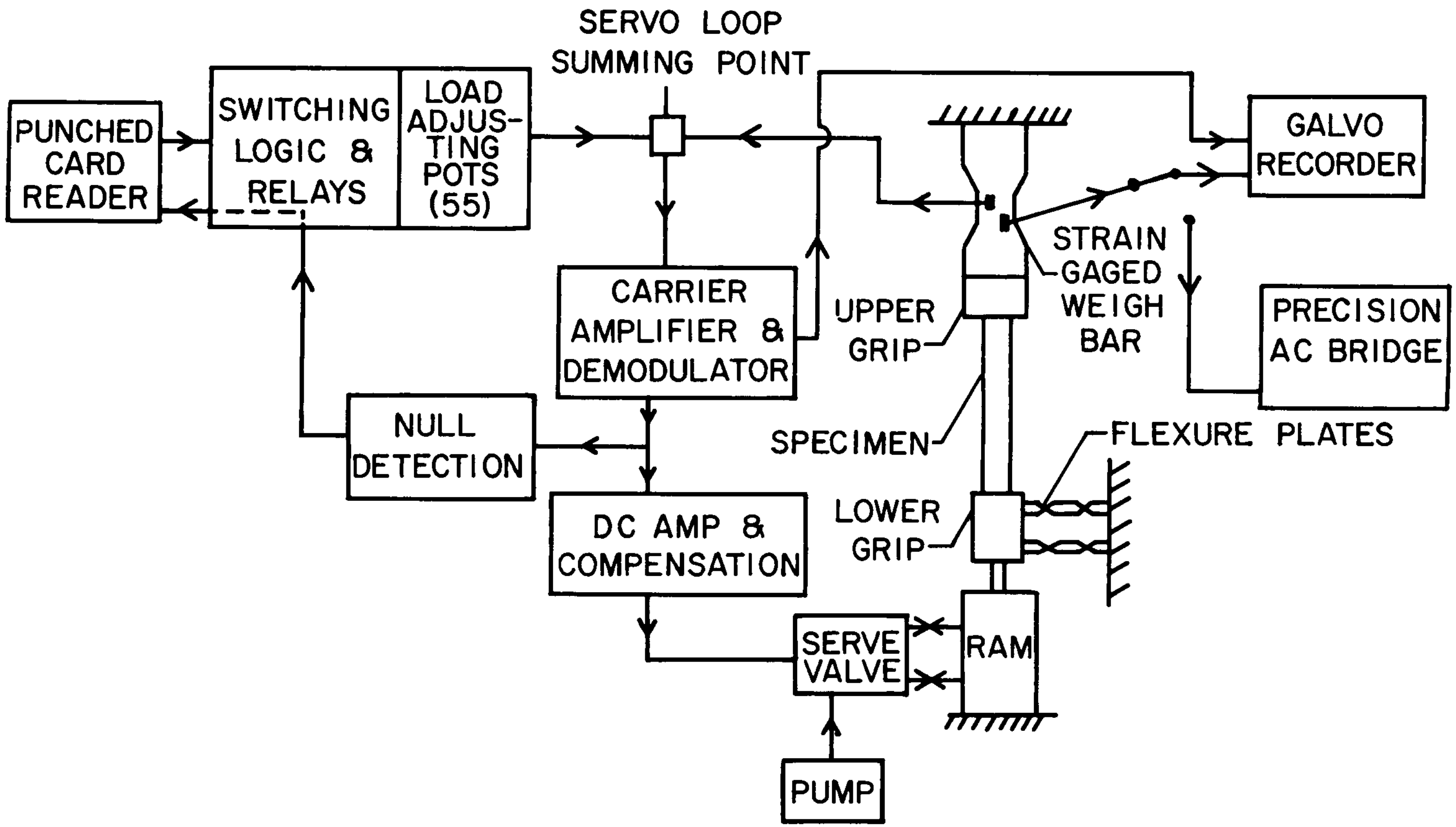

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Figure 10.- "Block" diagram of variable-amplitude testing machine.

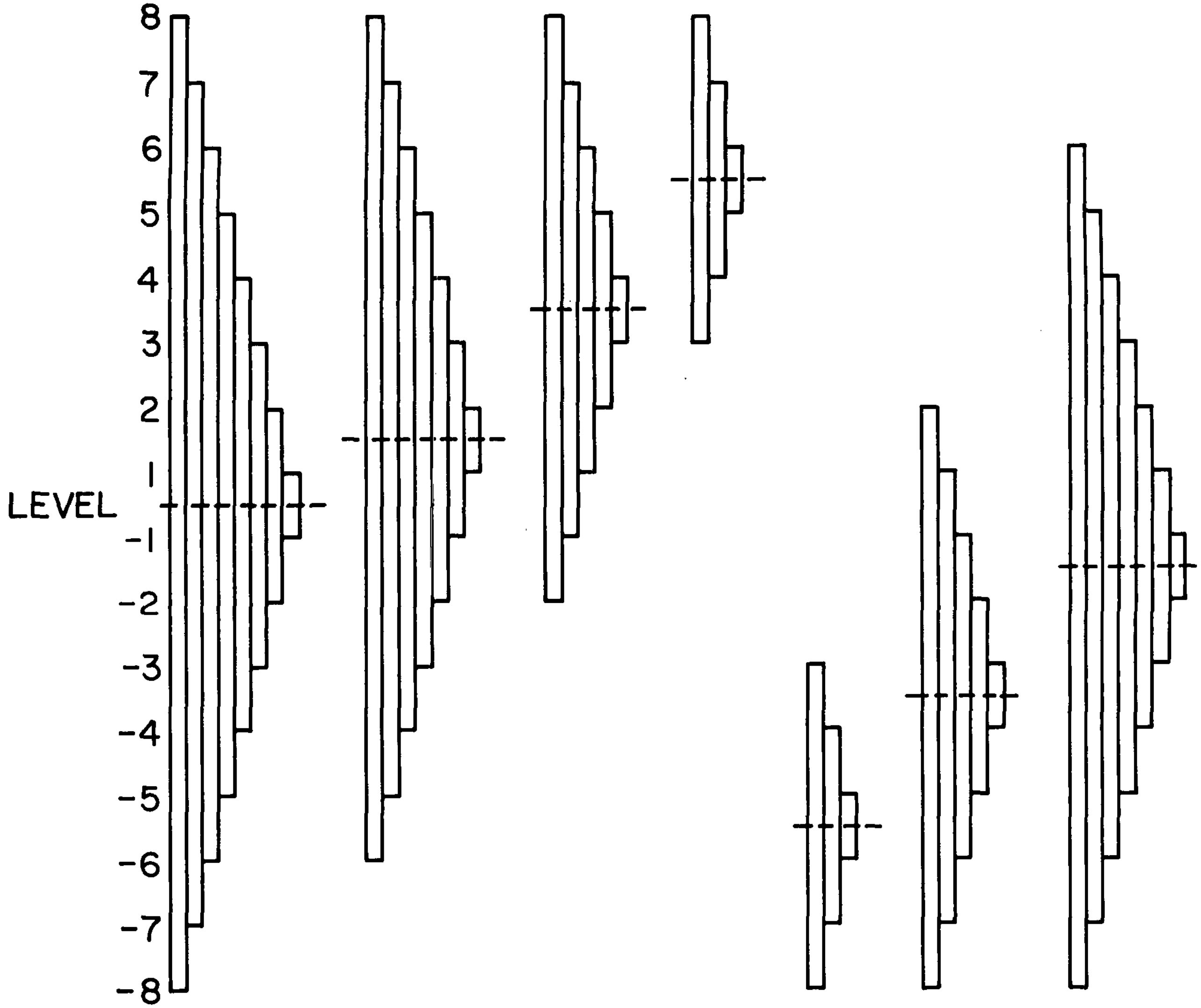

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Figure 11.- Schematic of 8-step variable mean block test.

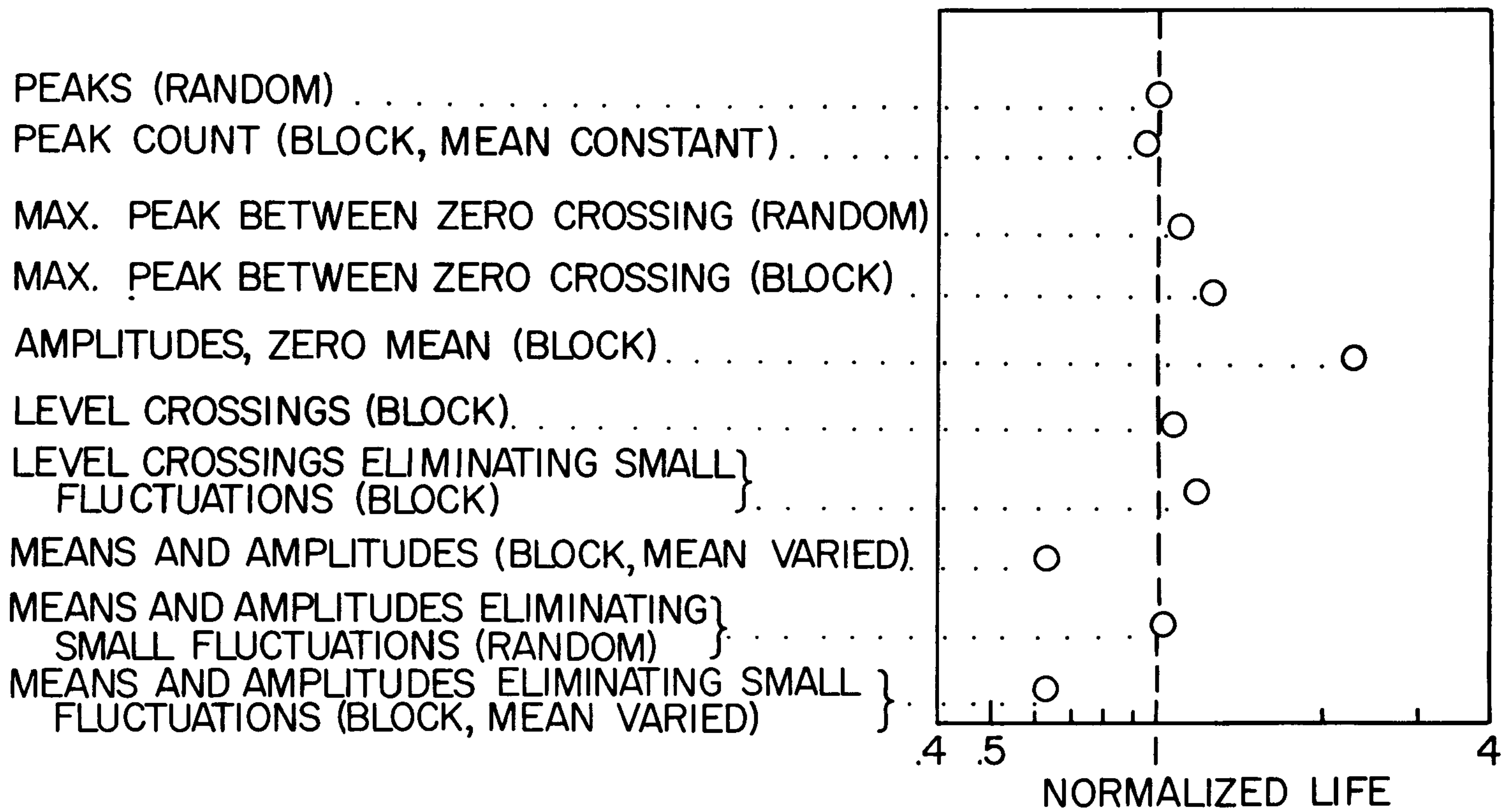

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Figure 12.- Normalized results of variable-amplitude fatigue tests.

NASA-Langley, 1963